Google Maps provide coordinates of latitude and longitude in decimal form to six places past the decimal point. For example, here are the coordinates for the Statue of Liberty provided by Google Maps: latitude 40.689261 longitude -74.044543. I obtained these coordinates by zooming in the street map to full zoom, then clicked on the center of the Statue of Liberty. A box then appeared with the coordinates near the bottom of the screen. If you try this, your numbers will probably be different than mine. One reason is that it is difficult to click in the exact center of an object. And when the coordinates are specified to six places past the decimal point, a difference of one in the last place is only a few inches on the surface of the Earth!

As I was thinking about these numbers, I became curious about the accuracy of the coordinates displayed on Google Maps. Are they really accurate to six places past the decimal point? The short answer is NO. We can start to investigate this by doing some calculations.

Due to its rotation, the Earth is not a true sphere (it bulges around the equator). Nevertheless, to a close approximation, we can treat our home as a true sphere. The Earth has an equatorial circumference of 24,901 miles (distance one time around the equator at sea level) and a meridional circumference of 24,812 miles (distance one time around the Earth, passing through both poles).

All lines of longitude pass through the poles, which makes this the easiest of the two coordinates to deal with mathematically. The longitude of -74.044543 circumscribes a line around the Earth that is 24,812 miles long. There are 360 degrees in a circle, therefore, one degree will equate to a distance of 68.922222 miles (24,812/360). Then 0.1 degree would be 6.892222 miles long. And 0.01 degree would be 0.689222 miles long. I think you can see where this is going: 0.001 degree would be 0.068922 miles. We can convert to feet by multiplying by 5,280 x 0.068922 = 363.91 feet. And 0.0001 degree would be 36.39 feet; 0.00001 would be 3.639 feet and finally, 0.000001 degree would be 0.3639 feet (or 4.37 inches). I have calculated the distance of 0.000001 degree on the surface of the Earth longitudinally and found that it is approximately 4.37 inches. Can we accept that Google Maps can plot a position to an accuracy of less than 5 inches on the surface of the Earth? It does not seem likely, and in fact, Google makes no claims to the accuracy of the coordinates provided. Nevertheless, it is helpful to know how small a distance 0.000001 degree equates to as we work with GPS coordinates and Google Maps.

We could stop here and not have to exert ourselves very much with a mathematical treatment. However, we should also make an appraisal of latitude. We are not going to get away with a simple math problem for latitude. Every line of latitude circumscribes the Earth with a different distance. In order to calculate the circumference of these circles, we need to use trigonometry. Even if you have not had trigonometry in school yet, I think you will be able to understand the math I am about to present to you.
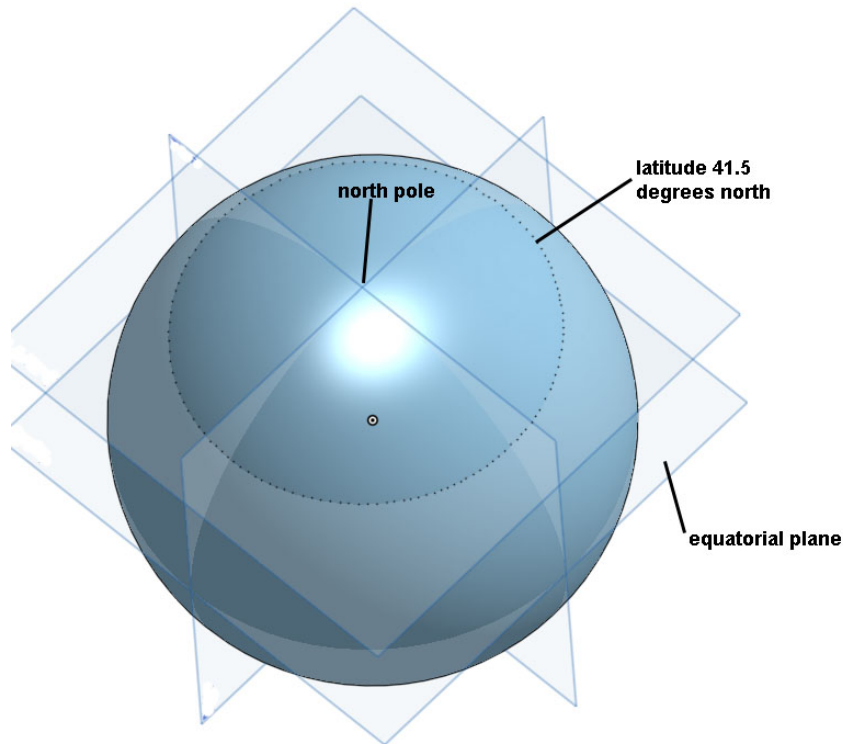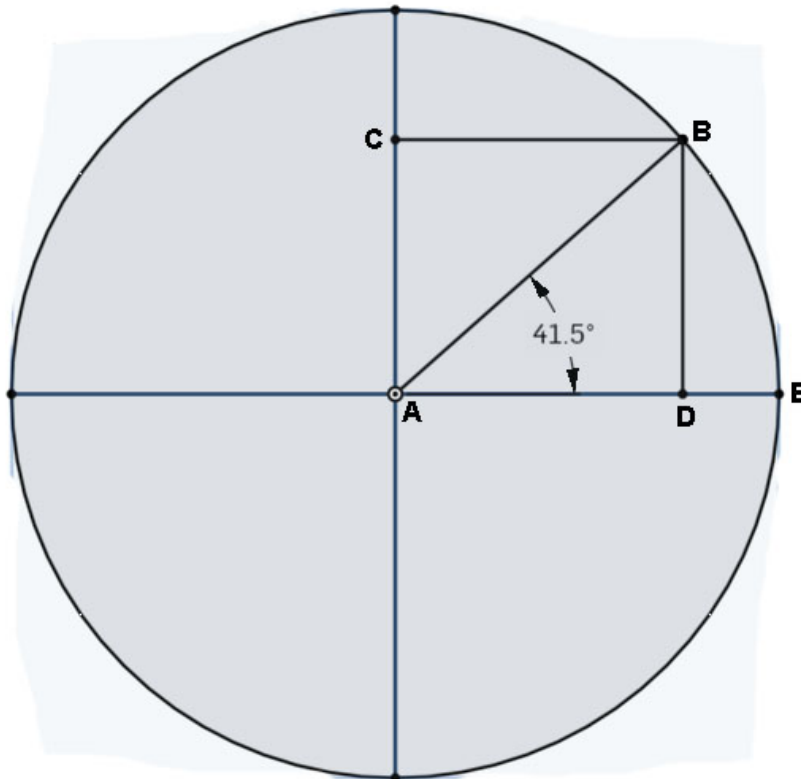
The latitude of Chesterland, Ohio is 41.5 degrees north, so let us take that as an example. In Figure 1, a dotted circle has been applied to the sphere representing Earth. The circle is positioned at a latitude of 41.5 degrees. I think you can see that this circle has a smaller circumference than the equator of the Earth. That means that each degree along this circle will contain less distance than one degree along the equator (or one degree along any line of longitude).

Figure 2 will help us do our calculations. The center of the Earth is marked by point A. Point B is a point on the surface of the Earth at latitude 41.5 degrees. The line AE is the equatorial radius of the Earth. The line CB is the radius of the Earth at 41.5 degrees latitude (that is, the radius that will serve to form a circle at latitude 41.5 degrees). If we can calculate the length of line CB, then we can calculate the circumference of the line circumscribing the Earth at latitude 41.5 degrees.

From the rules of geometry we know that the angle formed by points CBA is 41.5 degrees because CB is parallel to AE. We also know the length of the line AB (the radius of the Earth). Knowing these two things, we can calculate the

Figure 1 circumference at latitude 41.5

Figure 2 geometrical diagram

length of CB using a trigonometric function called cosine.

Triangle ACB is a right triangle (the angle ACB is 90 degrees). Line AB of this triangle is called the hypotenuse. And line CB is the line adjacent to the angle CBA. The formula for cosine is:

$$\cos \Theta = \text{adjacent/hypotenuse}$$

where **Ө** is the angle (*i.e.* degrees latitude)
adjacent is the length of line adjacent to the angle (it is one line that forms the angle)
hypotenuse is the length of the line opposite the 90 degree angle (in our case the radius of Earth)

With a calculator you can find the cosine of 41.5 degrees, which is 0.748956

The average radius of the Earth is 3,959 miles and we will use that figure.

Therefore:

$$0.748956 = \text{adjacent}/3{,}959$$

Rearranging the equation we get:

$$\text{adjacent} = 0.748956 \times 3{,}959 = 2965 \text{ miles}$$

Therefore, the radius of the circle for 41.5 degrees latitude is 2965 miles. We convert that to a circumference by multiplying by 2π, or 6.2832:

$$6.2832 \times 2965 = 18{,}629 \text{ miles}$$

Then we divide by 360 degrees:

$$18{,}629 / 360 = 51.75 \text{ miles per degree}$$

And if we multiply by 0.000001 we get distance per 0.000001 degree:

$$0.000001 \times 51.75 \text{ miles} = 0.00005175 \text{ miles}$$

or

$$5{,}280 \text{ feet per mile} \times 0.00005175 \text{ miles} = 0.273 \text{ feet or } 3.28 \text{ inches}$$

Now, if we compare the distance equivalent of 0.000001 degree at our latitude and longitude it is 3.28 inches in latitude and 4.37 inches in longitude. We should expect the length of latitude to be smaller and it is, but in the grander scheme of things, there really is not much difference between the two. They are both very small numbers! Keep in mind that the reference to latitude is actually distance in the east-west direction and longitude is distance in the north-south direction.